Subject SP2

CMP Upgrade 2023/24

CMP Upgrade

This CMP Upgrade lists the changes to the Syllabus, Core Reading and the ActEd material since last year that might realistically affect your chance of success in the exam. It is produced so that you can manually amend your 2023 CMP to make it suitable for study for the 2024 exams. It includes replacement pages and additional pages where appropriate.

Alternatively, you can buy a full set of up-to-date Course Notes / CMP at a significantly reduced price if you have previously bought the full-price Course Notes / CMP in this subject. Please see our 2024 *Student Brochure* for more details.

We only accept the current version of assignments for marking, *ie* those published for the sessions leading to the 2024 exams. If you wish to submit your script for marking but only have an old version, then you can order the current assignments free of charge if you have purchased the same assignments in the same subject in a previous year, and have purchased marking for the 2024 session.

This CMP Upgrade contains:

- all significant changes to the Syllabus objectives and Core Reading

- additional changes to the ActEd Course Notes and Assignments that will make them suitable for study for the 2024 exams.

1   Changes to the Syllabus

There have been a significant number of changes to how the syllabus objectives are expressed, but the underlying contents are basically unchanged. These changes have been made to the syllabus boxes in the relevant chapters.

Syllabus objectives 0 (principal terms) and 6 (solving problems) have been removed, but equivalent wording has instead been included within the wider Syllabus.

A copy of the new Syllabus can be found within the SP2 Study Guide, which is available to download from the **ActEd.co.uk** website: see Products, Study Guides (under 'Core Study Materials').

2   Changes to the Core Reading and ActEd text

This section contains all the *non-trivial* changes to the Core Reading and ActEd text.

Chapter 7

Section 3

The heading for Section 3.4 has been amended as follows:

Provision of capital and deferral of distribution

The Core Reading under the heading 'Revalorisation method' on page 16 has been amended as follows:

There is less scope for deferral of profit distribution with this method.

Summary

The first two headings have been renamed as 'Revalorisation method' and 'Contribution method'.

The last paragraph under the heading 'Contribution method' has been deleted and the penultimate paragraph has been amended as follows:

A terminal dividend may be given when the policy becomes a claim. This leads to some deferral of profit distribution.

Chapter 11

Question 11.7

The sixth bullet point in the question has been amended as follows:

- There is a fund management charge of 1% *pa* of the value of units, deducted monthly. Deductions are also made monthly from the value of units for the cost of life cover. The charges and deductions are variable in certain circumstances.

Solution 11.5

The overall heading for the solution has been changed to 'Product launch risks'. A new sub-heading has been added immediately below this of 'Investment risk'.

Chapter 13

Section 6.4

This section has been rewritten to show different approaches to allowing for the initial charge. Replacement pages are attached.

Question 13.3

The dates in the question have been moved on by a year so that it now refers to 2022 and 2023.

Solution 13.3

The solution has been rewritten to show different approaches to allowing for the initial charge. Replacement pages are attached.

Chapter 14

Solution 14.1

The second point has been amended as follows:

The model will project the expected cashflows arising in every future policy year, for any specified model point.   [½]

The second bullet point has been amended as follows:

- changes to any of the features of the model point (*eg* different ages, policy terms, *etc*).
                                                                                              [½]

Chapter 16

Solution 16.2

The overall heading for the solution has been changed to 'Considerations in deciding on product design'. A new sub-heading has been added immediately below this of 'Marketability and distribution'.

Chapter 21

Section 5

The five situations described on page 16 have been replaced with the following example:

Example

We can illustrate the impact on retained profit of experience being different to expected in the following example.

Consider a regular premium without-profits endowment.

If interest rates rise suddenly late on in the term of a without-profits endowment, then the matching bonds will fall in value and so the asset share drops. The realistic prospective value on the new higher best estimate interest rates also drops, so that the company still expects to make a profit.

However, if surrender values continue to be based on the original premium basis, then the surrender value will not fall in line with the lower value of the contract. The surrender value may be larger than the asset share so that the company now makes a loss.

So, using the original premium basis to calculate surrender values may become unsuitable over time. A possible solution to this problem would be to use the current premium basis to calculate surrender values.

Chapter 23

Solution 23.4

Further explanation has been added. Replacement pages are attached.

Chapter 24

Section 1

The first paragraph on page 10 has been updated as follows:

Excess of loss is an example of *non-proportional* reinsurance, because the <u>amount that the reinsurer pays is not defined as a proportion of the claim amount. Both</u> risk premium and original terms reinsurance are examples of *proportional* reinsurance types.

Chapter 31

Syllabus objectives

The syllabus objectives for this chapter have been deleted. The following has been added in its place:

There are no explicit syllabus objectives covered in this chapter. However, the Subject SP2 Syllabus indicates that:

Students will be expected to be able to apply knowledge and skills from across the syllabus topics to scenarios and questions proposed by the examiners and produce coherent solutions and actions, including:

- analysing complex problems in terms of actuarial, economic and financial factors to a level where appropriate analytical techniques may be used

- assessing the implications and relevance of such factors, integrating the results into a coherent whole

- evaluating the results critically in a wider context, drawing appropriate conclusions

- proposing solutions and actions, or a range of possible solutions and actions, based on this evaluation.

Section 0

The first two paragraphs of Core Reading have been replaced by the following ActEd and Core Reading text:

There is no explicit syllabus objective for this chapter.

However, as part of their preparation for the assessment, candidates must demonstrate knowledge and understanding of the subject as a whole, in order to apply and produce solutions to problems relating to life insurance as follows:

- **Combine ideas across the chapters in the Subject SP2 Core Reading, and apply them to the scenarios proposed by the examiners.**

- **Analyse hypothetical examples and scenarios in relation to the application of life insurance principles.**

- **Propose solutions and actions that are appropriate to the given context and balance benefit against inherent cost, with justification where required.**

- **Suggest possible reasons why certain actions have been chosen.**

- **Assess the implications of actions within a given scenario.**

- **Discuss the advantages and disadvantages of suggested actions, taking into account the perspectives of different stakeholders.**

Examiners will expect candidates to be able to apply the knowledge and understanding they have developed through the study of the Core Reading for this subject to produce coherent solutions and actions in relation to the financial management of a life insurance company.

Chapter 32

Syllabus objectives

The syllabus objectives for this chapter have been deleted. The following has been added in its place:

There are no explicit syllabus objectives covered in this chapter. However, the Subject SP2 Syllabus states that:

Students should ensure that they … have an understanding of the principal terms used in life insurance.

3 Changes to the X Assignments

Overall

There have been minor changes throughout the assignments, including changes to mark allocations.

More significant changes are listed below.

Assignment X1

Solution X1.2

The second heading has been changed to 'Expenses and inflation'.

The first point below the heading of 'Capital strain' has been replaced by the following:

There is a risk that new business volumes are higher than expected leading to additional capital strain. [½]

However, initial reserves and solvency capital should not be too onerous as charges are reviewable … [½]

Solution X1.4

The final point has been updated as follows:

So the company is at risk from more new business strain if new business volumes are higher than expected. [½]

Solution X1.5

The eleventh point has been updated as follows:

The cohort asset share of the term assurance will not be adjusted for the profits on without-profits business or withdrawals. [½]

The final point on page 7 has been updated as follows:

… as it has the higher sum assured and the lower asset share (and therefore the higher excess of sum assured over asset share). [½]

Solution X1.7

The marks awarded for the third point in part (i)(b) have changed as follows:

… in order to cover (often explicitly) the various costs incurred – such as investment costs, selling expenses / commissions, and the cost of meeting claims. [1]

Assignment X2

Question X2.5

The bullet points have been updated as follows:

- At least 60% of investment must be in government bonds.

- Remaining investment must be in quoted equity shares, <u>quoted fixed-interest corporate bonds</u>, or cash.

- No overseas investment is allowed.

- No more than 1% of a company's total assets may be invested in the equity or <u>bonds</u> of an individual company.

Solution X2.1

The last point in the solution to part (ii) has been updated as follows:

Probably the most important change would be an increase in the average policy size, which <u>could result in being able to apply</u> lower charges <u>(if expressed as a percentage of fund and/or premium)</u> for the same profitability. [1]

Solution X2.6

The second point under the heading 'New business volume' has been updated as follows:

<u>The charges taken per policy will, however, not depend on the number of policies sold</u>, so there could be a significant risk from not selling enough policies. [½]

Solution X2.7

The final point has been split so that more marks are available as follows:

Failure of management to operate the systems effectively is a significant risk. [½]

The company will not be familiar with the process of mortality-experience monitoring and the associated charge and premium reviews. [½]

Assignment X3

Solution X3.3

The eighth point in the solution to part (i) has been updated as follows:

The model should allow for the potential cashflows from any <u>mortality</u> options included in the contracts, ... [½]

Solution X3.4

The fifth point under 'Step 4' has been updated as follows:

- … allowing for non-unit cashflows, *ie* charges plus investment return less <u>expenses and the cost of any death benefits in excess of the unit fund</u>   [½]

Solution X3.7

In the solution to part (ii), the eighth point has been moved so that it comes before the twelfth point, and the twelfth point has been rewritten, so that the new solution is:

A (monetary-related) surrender penalty could be introduced in years 1 and 2, equal to (at least) the amount of unrecouped allocation charges.   [½]

<u>The company may then be able to hold</u> negative non-unit reserves in those two <u>years</u>.   [½]

Assignment X4

Question X4.5

The date has been rolled forward a year to 2023.

Solution X4.1

The first two points under the heading 'Surrender values' have been updated as follows:

<u>Requiring policy reserves to</u> be less than any guaranteed surrender <u>value is</u> appropriately prudent.   [½]

<u>Allowing some</u> negative <u>reserves may</u> be imprudent <u>on its own</u> …   [½]

The final point under the heading 'Surrender values' has been updated as follows:

… because for a small company it could represent a significant portion of the reserves, for a large company hardly anything <u>(although it is difficult to say without knowing how much a $ is worth)</u>.   [½]

The first point under the heading 'Unit-linked' has been updated as follows:

<u>Unit-linked business will soon start to be issued</u>, so such business needs to be catered for.   [½]

Solution X4.2

The marks awarded for the eighth point have changed as follows:

However, it may be possible (depending on the purpose of the valuation and the regulations that apply to it) to take credit for the illiquidity premium in corporate bonds and thereby use a higher discount rate than the risk-free rate.   [1]

Solution X4.7

The second and fifth bullet points in the solution to part (i) have changed as follows:

- It is simple to <u>do</u>.   [½]

- It is capable of being clearly <u>explained</u> / documented.   [½]

Solution X4.9

In the second bullet point list in the solution to part (ii), the first bullet point has changed as follows:

investment return earned on reserves <u>(and on cashflows to the end of each projection period)</u>
                                                                                                                                                      [½]

Assignment X5

Solution X5.2

The last two points have been updated as follows:

If model <u>points</u> are being used then group the data together, maybe by premium size, age, term, expired duration, different charging structures and so on.   [½]

For each group of data decide upon a suitable 'average' model <u>point</u>.   [½]

Solution X5.4

The tenth point has been updated as follows:

Quota share reinsurance would be used to reduce the exposure to pricing / <u>parameter</u> risks, if needed.   [½]

Solution X5.8

The final point in part (d) has been split so that more marks are available as follows:

People may lapse rather than repay the discount.   [<u>½</u>]

Such lapses may also be selective.   [½]

Solution X5.9

The second and third points have been updated as follows:

<u>So using</u> reinsurance <u>allows large cases to</u> be accepted <u>by reinsuring</u> the excess above a certain <u>level</u>.   [½]

Reinsuring a higher proportion of larger risks (by using individual surplus, say) will reduce claims volatility …   [<u>½</u>]

… and help produce more stable profits …   [<u>½</u>]

Assignment X6

Solution X6.1

The second point in part (b) has been updated as follows:

However, they could do some <u>maintenance</u> admin and so this part could be included as servicing costs.   [½]

Solution X6.6

In part (iii), the last point under the heading 'First, what does a lower persistency assumption imply for prudence?' has been expanded into two points as follows:

This is unlikely <u>for endowment assurances</u>, unless there are no PRE / marketing pressures on the <u>company's surrender values …</u>   [½]

<u>… but may be possible for term assurance lapses where the asset share is likely to be highest mid-term</u>.   [½]

In part (iii), the third point under the heading 'Next, is extra prudence wanted?' has been replaced by the following two points:

<u>For example, if reserves are to be calculated using a market-consistent basis, then the risk margin component might be increased instead of changing the best estimate persistency assumption.</u>   [½]

For instance, if these prescribe 'choose a 0% or 100% withdrawal rate for each product class (or policy) depending on which is more prudent' (which has been <u>used in some jurisdictions</u>), then the whole question is immaterial.   [½]

In part (iii), the penultimate point under the heading 'Next, is extra prudence wanted?' has been updated as follows:

If there is some doubt about the validity of the company's own assumptions, it is better to move a bit more in the direction of prudence, especially if rates are guaranteed. So the MD's suggestion could be reasonable here (if higher <u>withdrawal rates are</u> more prudent for the products concerned), assuming that this does not put the company out of the market.   [1]

4 Other tuition services

In addition to the CMP you might find the following services helpful with your study.

4.1 Study material

We also offer the following study material in Subject SP2:

- Flashcards

- Revision Notes

- Sound Revision

- ASET (ActEd Solutions with Exam Technique) and Mini-ASET

- Mock Exam and AMP (Additional Mock Pack).

For further details on ActEd's study materials, please refer to the 2024 *Student Brochure*, which is available from the ActEd website at **ActEd.co.uk**.

4.2 Tutorials

We offer the following (face-to-face and/or online) tutorials in Subject SP2:

- a set of Regular Tutorials (lasting a total of three days)

- a Block Tutorial (lasting three full days)

- an Online Classroom.

For further details on ActEd's tutorials, please refer to our latest *Tuition Bulletin*, which is available from the ActEd website at **ActEd.co.uk**.

4.3 Marking

You can have your attempts at any of our assignments or mock exams marked by ActEd. When marking your scripts, we aim to provide specific advice to improve your chances of success in the exam and to return your scripts as quickly as possible.

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4.4   Feedback on the study material

ActEd is always pleased to get feedback from students about any aspect of our study programmes. Please let us know if you have any specific comments (*eg* about certain sections of the notes or particular questions) or general suggestions about how we can improve the study material. We will incorporate as many of your suggestions as we can when we update the course material each year.

If you have any comments on this course, please send them by email to **SP2@bpp.com**.

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These conditions remain in force after you have finished using the course.

Initial charges

When units are allocated to policyholders, companies may want to make an additional charge as a contribution to meeting other management expenses and any commission payments and as a contribution to profits. This initial charge is an addition to the appropriation price.

The offer price could then be taken as equal to the appropriation price plus the initial charge and the bid price as equal to the appropriation price.

This initial charge may also be referred to as a 'bid-offer spread'.

Rounding

It is normal to quote prices to a certain number of decimal places. This could be done by rounding the offer price up and the bid price down, for example, which rounds in favour of the insurance company. Alternatively, rounding the offer price down and the bid price up rounds in favour of the customer.

Actual practice varies, although there has been a general trend away from systematic rounding against the customer.
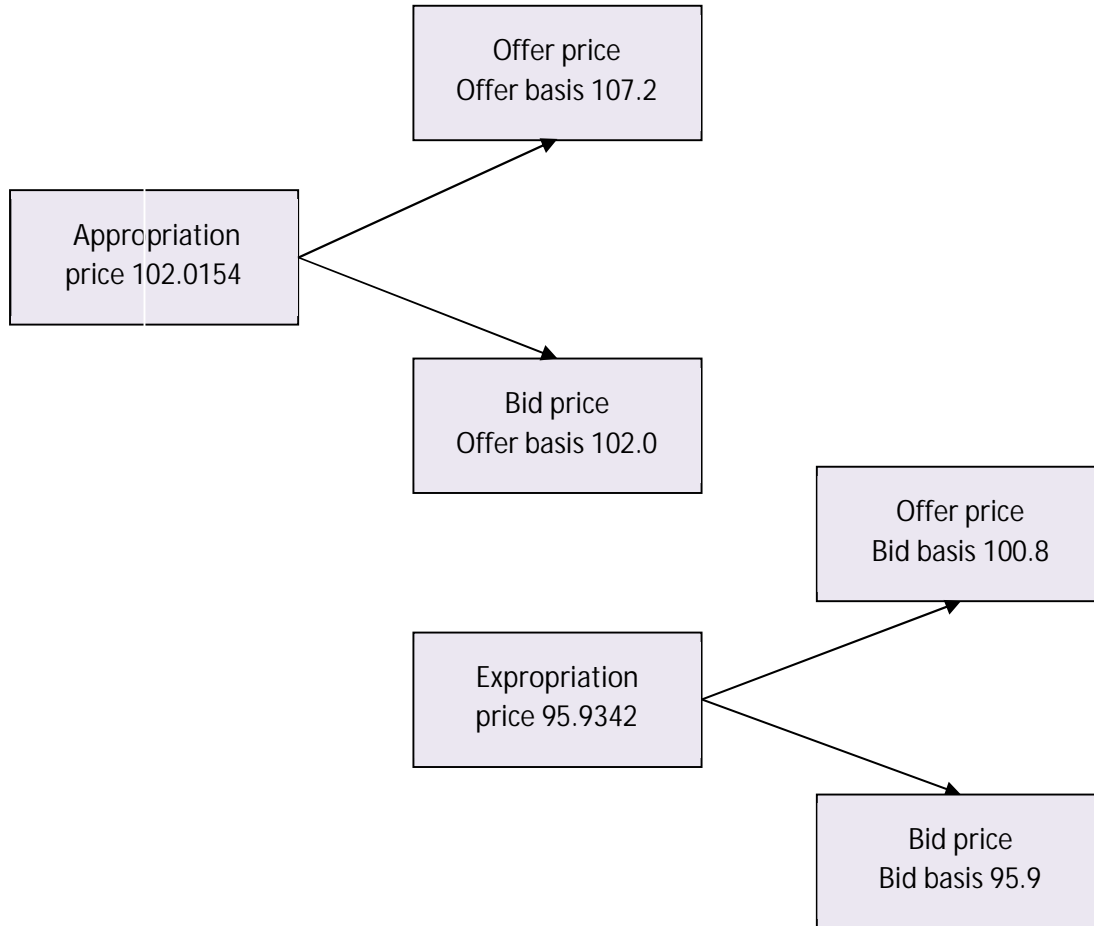
6.3 Offer and bid prices when pricing on a bid basis

Consider next the situation, again with daily pricing, where for a particular fund on a particular day the number of units being surrendered exceeds that being allocated.

As fund units must be cancelled, pricing should be on a bid basis, with offer and bid prices being derived from the expropriation price. The rest of the process follows exactly that for deriving offer and bid prices when pricing on an offer basis, but using the expropriation price.

6.4   Numerical example

The diagram below gives one hypothetical example of how the various different prices might compare. An explanation is given on the next page. Prices are in pence per unit.

```
                              ┌─────────────────────┐
                              │     Offer price     │
                              │  Offer basis 107.2  │
                              └─────────────────────┘
                                       ▲
┌─────────────────────┐               
│    Appropriation    │──────────┐    
│   price 102.0154    │          ▼    
└─────────────────────┘   ┌─────────────────────┐
                          │      Bid price      │
                          │  Offer basis 102.0  │
                          └─────────────────────┘

                                         ┌─────────────────────┐
                                         │     Offer price     │
                                         │   Bid basis 100.8   │
                                         └─────────────────────┘
                                                  ▲
                          ┌─────────────────────┐ 
                          │    Expropriation    │────────┐
                          │    price 95.9342    │        ▼
                          └─────────────────────┘ ┌─────────────────────┐
                                                  │      Bid price      │
                                                  │   Bid basis 95.9    │
                                                  └─────────────────────┘
```

These prices were derived from the following example method:

Offer basis

For an expanding fund, the prices are calculated as follows:

- take the market offer price plus expenses (assume this value is 102.0154p per unit) – this is the appropriation price

- round to the lower 0.1p

- bid price on an offer basis 102.0p (expanding fund)

- multiply the appropriation price by 1.05 (assume a 5% initial charge)

- take the answer to the higher 0.1p

- offer price on an offer basis 107.2p (expanding fund).

In the calculation above, we have assumed that the initial charge (bid-offer spread) is allowed for by setting the offer price to be 5% higher than the appropriation price. It is equally acceptable in the exam to assume that the initial charge is allowed for by setting the offer price so that the appropriation price is 5% lower than the offer price (in this case we would divide the appropriation price by 0.95 and obtain an offer price of 107.4).

Bid basis

Similarly, for a contracting fund:

- take the market bid price less expenses (assume this is 95.9342p per unit) – this is the expropriation price

- take the answer to the lower 0.1p

- bid price on a bid basis 95.9p (contracting fund)

- multiply the expropriation price by 1.05

- round to the higher 0.1p

- offer price on a bid basis 100.8p (contracting fund).

Again in the calculation above, we have assumed that the initial charge is allowed for by setting the offer price to be 5% higher than the expropriation price. If the initial charge is allowed for by setting the offer price so that the expropriation price is 5% lower than the offer price then we would divide the expropriation price by 0.95 and obtain an offer price of 101.0.

6.5   Offer basis or bid basis?

In Sections 6.2 and 6.3 above, the notes suggest that companies decide the pricing basis taking account of just that day's cashflow.

That is, on days when the company is a net creator of units, it will price on an offer basis, and on days when it is a net eliminator of units it will price on a bid basis.

However, in practice, companies are more likely to use a 'broad equity' approach under which the basis is only changed if there is a significant cashflow movement against the existing basis *eg* if there is a significant net cash inflow for a fund currently being priced on a bid basis. This approach provides broad equity between different unit-holders and reduces price volatility.

Many companies will use a management box to minimise changes to the basis. For instance, suppose we have a net 'order' for one day of a sale of 2,300 units, and the company has a box of 13,000 units for that fund. If the company has been pricing on an offer basis for the last few days then it would probably add the units to the box (ie buy the units for its own account), rather than redeem them. However, this would be subject to the box not then exceeding some pre-agreed limit.

Chapter 13 Solutions

13.1   Unit pricing

In general unit pricing will follow the basic equity principle: that the interests of unit-holders not involved in a unit transaction should not be affected by that transaction.

The pricing will depend on whether the company is creating or cancelling units in the fund.

This situation might be looked at daily or, more likely, over a longer period to avoid changing the basis too frequently.

If the company is creating units then it values units on an 'offer basis'.

Under an offer basis, offer and bid prices for units are derived from the appropriation price.

The appropriation price is the amount of money that the company should put into the fund in respect of each unit created in order that the interests of existing unit-holders are not affected.

It can be calculated as the net asset value of the fund on an 'offer basis' divided by the number of units in the fund (prior to the creation of the units concerned).

This net asset value is the offer price value per unit of the fund's underlying assets, adjusted in respect of the fund's current assets and liabilities, accrued income, and acquisition dealing expenses.

The expropriation price is the amount of money that the company should remove from the fund in respect of each unit in order that the interests of existing unit-holders are not affected.

Calculated as net asset value on a 'bid basis' divided by number of units ...

... where net asset value is the bid price value of units, adjusted for current assets and liabilities, accrued income, and sale dealing expenses.

If the company wants to avoid switching from one basis to the other too frequently then it will probably need to create a 'management box'.

Given the basis (offer or bid), the company will calculate the unit price as:

*   offer price: the appropriation price (on an offer basis) or expropriation price (on a bid basis), plus initial charges

*   bid price: the appropriation / expropriation price.

The above prices will need to be rounded. Some companies round the offer price up and the bid price down. However, it is now increasingly common for companies to round to the nearest decimal place.

13.2   (i)   Management box

The 'management box' for a fund consists of units that have been created but are not owned by policyholders, at any point in time. These units are therefore owned by the life insurance company itself.   [1]

(ii)   Advantages and disadvantages

The primary advantage of the management box is that it enables the company to maintain a steady offer or bid basis, rather than having to change too frequently the basis on which it prices units.   [1]

This achieves 'broad equity' between different unit-holders (as the basis is only changed if there is a significant cashflow movement against the existing basis) and avoids unnecessary and artificial volatility in published unit prices.   [1]

By using a box, unnecessary creation and cancellation of units is avoided …   [½]

… and so the associated transaction costs of buying and selling assets in the market is avoided too.   [½]

The main disadvantage to the company is that it will be exposed to the investment risk of holding a large number of units for its own account.   [1]

Although with careful management it is a potential source of additional profit.   [½]

There will be expenses related to managing the box.   [½]

In addition there may be problems with control – setting down guidelines as to how large a box is maintained, what to do in the event of severe market movements or large unit purchases / encashments.   [1]
                                                                                      [Maximum 4]

13.3   We will need the offer price on both bases at 1.7.2022, and the bid price on both bases at 1.7.2023. Unit prices are rounded to the nearest penny, as stated in the question.

1.7.2022

The expropriation price per unit is: $\dfrac{0.98 \times 1,137.5}{325} = 3.43$

The offer price on the bid basis is therefore: $3.43 \times 1.05 = 3.60$

(Alternatively, the offer price on the bid basis can be calculated as: $\dfrac{3.43}{0.95} = 3.61$)

The appropriation price per unit is: $\dfrac{1.03 \times 1,183}{325} = 3.7492$

The offer price on the offer basis is therefore: $3.7492 \times 1.05 = 3.94$

(Alternatively, the offer price on the offer basis can be calculated as: $\dfrac{3.7492}{0.95} = 3.95$)

1.7.2023

The expropriation price per unit is: $\dfrac{0.98 \times 1,794}{460} = 3.822$

so the bid price on the bid basis is 3.82 after rounding.

The appropriation price per unit is: $\dfrac{1.03 \times 1,856.8}{460} = 4.1576$

so the bid price on the offer basis is 4.16 after rounding.

The returns are then calculated as follows:

(a)   Bid / bid basis

Number of units purchased on 1.7.2022 $= \dfrac{1,000}{3.60} = 277.78$ (after rounding)

Cash value at 1.7.2023:

$277.78 \times 3.82 = 1,061.12$

ie a return of 6.1%.

(If the alternative calculation for the offer price is used then:

Number of units purchased on 1.7.2022 $= \dfrac{1,000}{3.61} = 277.01$ (after rounding)

Cash value at 1.7.2023:

$277.01 \times 3.82 = 1,058.18$

ie a return of 5.8%.)

(b)   Bid / offer basis

Number of units purchased on 1.7.2022 = 277.78 from (a).

Cash value at 1.7.2023:

$277.78 \times 4.16 = 1,155.56$

ie a return of 15.6%.

(If the alternative calculation for the offer price is used then:

Number of units purchased on 1.7.2022 = 277.01 from (a).

Cash value at 1.7.2023:

$$277.01 \times 4.16 = 1,152.36$$

ie a return of 15.2%.)

(c)   Offer / bid basis

Number of units purchased on 1.7.2022 $= \dfrac{1,000}{3.94} = 253.81$

Cash value at 1.7.2023:

$$253.81 \times 3.82 = 969.55$$

ie a return of –3.0%.

(If the alternative calculation for the offer price is used then:

Number of units purchased on 1.7.2022 $= \dfrac{1,000}{3.95} = 253.16$

Cash value at 1.7.2023:

$$253.16 \times 3.82 = 967.07$$

ie a return of –3.3%.)

(d)   Offer / offer basis

Number of units purchased on 1.7.2022 = 253.81 from (c).

Cash value at 1.7.2023:

$$253.81 \times 4.16 = 1,055.85$$

ie a return of 5.6%.

(If the alternative calculation for the offer price is used then:

Number of units purchased on 1.7.2022 = 253.16 from (c).

Cash value at 1.7.2023:

$$253.16 \times 4.16 = 1,053.15$$

ie a return of 5.3%.)

From the Tables we see that $\ddot{a}_{60}^{@4\%} = 14.134$ and $\ddot{a}_{60}^{@6\%} = 11.891$. Since discounting at a higher rate of interest reduces the present value, it follows that $\ddot{a}_{60}^{@8\%} < \ddot{a}_{60}^{@6\%}$. So the guarantee will only bite if $i = 4\%$.

We can now work out the values of L, conditional on the value of i. These are:

For $i = 0.04$:

$$L = 15,000\,\ddot{a}_{60}^{(12)@\,4\%} = 15,000\left(14.134 - \frac{11}{24}\right) = \pounds205,135$$

For $i > 0.04$ (that is, for $i = 6\%$ or $i = 8\%$):

$$L = \pounds200,000$$

as explained above.

The probability of interest, that is the probability that $A < L$, is therefore also conditional on i. We shall write this conditional probability as:

$$P\left(A < L \,\middle|\, i\right)$$

So:

$$P\left(A < L \,\middle|\, i = 4\%\right) = P\left(A < 205,135\right)$$

$$= \Phi\left(\frac{205,135 - 250,000}{50,000}\right)$$

$$= \Phi\left(-0.8973\right) = 1 - \Phi\left(0.8973\right)$$

$$= 0.1848$$

Similarly:

$$P\left(A < L \,\middle|\, i > 4\%\right) = P\left(A < 200,000\right)$$

$$= \Phi\left(\frac{200,000 - 250,000}{50,000}\right)$$

$$= \Phi\left(-1\right) = 1 - \Phi\left(1\right)$$

$$= 0.15866$$

Finally, our overall (unconditional) probability that $A < L$ is:

$$\sum_i P(A < L | i) \times P(i)$$

$$= P(A < L | i = 4\%) \times P(i = 4\%) + P(A < L | i > 4\%) \times P(i > 4\%)$$

$$= 0.1848 \times 0.25 + 0.15866 \times 0.75 = 0.1652 \, .$$

23.4   We calculate the single premium payable for the standard two-year contract using normal mortality (AM92 Ultimate) and 7% pa interest as follows:

$$P_0 = 50,000 \, [q_{50}v + (1 - q_{50}) \times q_{51}v^2]$$

$$= 50,000 \, [0.002508v + (1 - 0.002508) \times 0.002809v^2]$$

$$= 239.56$$

The premium paid at the start of the second year by the option-takers is calculated on standard terms, so again uses ultimate mortality:

$$P_1 = 25,000 \times q_{51}v$$

$$= 25,000 \times 0.002809v$$

$$= 65.63$$

We can calculate the value of the benefits by breaking them down into three parts: the benefits received by all policyholders in the first year, the benefits received in the second year by those that do not take up the option and the benefits received in the second year by those that do take up the option.

30% of the surviving policyholders exercise the option and are assumed to have double the rate of mortality of those that do not take up the option (this higher rate of mortality is only assumed to apply after exercising the option). So, the total value of the benefits is:

$$PV = 50,000 \times q_{50}v + 0.7 \times 50,000 \times (1 - q_{50})q_{51}v^2 + 0.3 \times 75,000 \times (1 - q_{50}) \times 2q_{51}v^2$$

$$= 50,000 \times 0.002508v + 50,000 \times 0.00196137v^2 + 75,000 \times 0.00168117v^2$$

$$= 312.98$$

The total value of the premiums is:

$$239.56 + 0.3 \times 65.63 \times (1 - q_{50})v = 239.56 + 65.63 \times 0.2992476v = 257.91$$

The value of the option is the total value of the benefits less the total value of the premiums paid on standard terms. So the additional single premium required to cover the cost of the option is:

$$312.98 - 257.91 = £55.07$$